**\*\*Muundo na muda vinaonekana vizuri!\*\***

**Uhusiano wa Euler Uthibitisho - manukuu**

1

00:00:28,000 --> 00:00:33,800

Sasa kwa kuwa umejadili kidogo

kujua jinsi uhusiano huu ni wa kweli,

2

00:00:33,900 --> 00:00:41,000

uhusiano huu F-A+S=2,

Ninapendekeza uionyeshe.

3

00:00:42,000 --> 00:00:45,000

Uhusiano huu unaitwa

Uhusiano wa Euler,

4

00:00:45,500 --> 00:00:49,000

baada ya mwanahisabati Leonhard Euler;

5

00:00:49,500 --> 00:00:52,900

na kwa hivyo tutathibitisha fomula hii kwa

kujirudia.

6

00:00:53,000 --> 00:00:56,900

Badala ya kukuonyesha kwa undani nini

mawazo ya kurudia ni nini,

7

00:00:57,000 --> 00:01:01,500

Ninapendekeza uonyeshe fomula hii

na michoro

8

00:01:01,600 --> 00:01:05,000

na utaona hoja kwa

kujirudia kidogo nje.

9

00:01:08,000 --> 00:01:09,900

Katika hoja ya kujirudia,

10

00:01:10,000 --> 00:01:14,900

ni lazima kwanza kuonyesha kwamba mali

tunachotaka kuonyesha

11

00:01:15,000 --> 00:01:26,000

(kwa hivyo mali yetu itakuwa S-A+F=2

kwa grafu yoyote iliyopangwa),

12

00:01:27,000 --> 00:01:30,000

ni kweli katika kesi ya awali.

13

00:01:30,800 --> 00:01:33,900

Na kwa upande wetu, kesi ya awali ni

kesi ambapo kuna kingo sifuri.

14

00:01:34,000 --> 00:01:38,900

Kesi iliyo na kingo za sifuri inalingana na

grafu ambapo kuna vertex moja tu

15

00:01:39,000 --> 00:01:40,500

(hapa unayo vertex).

16

00:01:40,600 --> 00:01:44,000

Ni grafu inayoruhusiwa kabisa:

ameunganishwa,

17

00:01:44,100 --> 00:01:49,000

tangu kwenda kutoka kwenye kipeo hiki hadi

mkutano huu, hakuna haja ya kufanya;

18

00:01:49,100 --> 00:01:51,500

na unaona kwamba hakuna kingo

ambayo inakatiza,

19

00:01:51,600 --> 00:01:54,000

kwani ipo kirahisi

hakuna makali.

20

00:01:56,000 --> 00:02:03,000

Tunathibitisha kuwa kwa grafu hii, kuna a

kipeo (kwa hivyo S=1), ukingo wa sifuri (kwa hivyo A=0),

21

00:02:03,100 --> 00:02:07,000

na uso, ambao ni uso wa nje

(kwa hivyo F=1).

22

00:02:07,100 --> 00:02:14,900

Na ikiwa unafanya hesabu,

S-A+F = 1-0+1 = 2.

23

00:02:15,000 --> 00:02:18,000

Kwa hivyo uhusiano wa Euler ni kweli kwa hili

kesi hii.

24

00:02:18,100 --> 00:02:22,000

Hii ndio kesi ya kwanza, na huu ndio mwisho

ya uanzishaji wa kujirudia.

25

00:02:23,000 --> 00:02:25,900

Sasa twende

hatua ya kurudia.

26

00:02:26,000 --> 00:02:28,900

Hii inajumuisha kudhani kuwa mali hiyo

tunachotaka kuonyesha

27

00:02:29,000 --> 00:02:33,400

ni kweli kwa baadhi ya A.

Mali tunayotaka kuonyesha

28

00:02:33,500 --> 00:02:38,100

ni uhusiano wa Euler (S-A+F=2),

na tuseme hivyo

29

00:02:38,150 --> 00:02:42,900

uhusiano huu wa Euler ni kweli kwa wote

grafu ambazo zina chini ya kingo 12,

30

00:02:43,000 --> 00:02:45,400

kwa hivyo ambayo ina kingo 12 au chini.

Na sasa,

31

00:02:45,450 --> 00:02:47,900

tuseme tunayo grafu iliyo na kingo 13.

32

00:02:48,000 --> 00:02:51,000

Hapa, ukihesabu idadi ya kingo,

kuna pembe 13.

33

00:02:52,000 --> 00:02:55,900

Hivyo ni jinsi gani sisi kwenda kuonyesha kwamba graph hii

inatosheleza uhusiano wa Euler?

34

00:02:56,000 --> 00:03:01,000

Ninapendekeza tu ufute

mwamba huu.

35

00:03:02,000 --> 00:03:07,000

Tunapata nini kama grafu?

Grafu inayotokana ni hii.

36

00:03:08,000 --> 00:03:12,000

Unaona kwamba tunapata grafu mpya,

ambayo sasa ina kingo 12.

37

00:03:12,500 --> 00:03:25,000

Hapa, A'=A-1.

Hapa, tulikuwa na A=13, kwa hivyo A'=12.

38

00:03:25,500 --> 00:03:29,000

Nini ilikuwa na athari ya kufuta hii

mfupa wa samaki?

39

00:03:30,500 --> 00:03:34,400

Tunaweza kuona wazi kwamba idadi ya wima haina

haijasogezwa: hakuna vertex iliyofutwa,

40

00:03:34,500 --> 00:03:41,000

kwa hivyo S' ni sawa na nambari

ya vipeo vya awali, kwa hivyo S'=S.

41

00:03:44,500 --> 00:03:49,000

Lakini sasa unaona nambari hiyo

uso umebadilika!

42

00:03:50,000 --> 00:03:55,000

Hapa unaona kwamba makali ya awali yalikuwa

kati ya pande mbili.

43

00:03:55,500 --> 00:04:02,000

Hapa kulikuwa na uso upande mmoja wa

mwamba, na kulikuwa na mwingine.

44

00:04:02,200 --> 00:04:05,000

Kwa hiyo walikuwa pande mbili tofauti.

45

00:04:05,100 --> 00:04:12,000

Sasa, hapa, nyuso hizi mbili zimeunganishwa na

form one tu.

46

00:04:12,500 --> 00:04:17,400

Kwa hiyo unaona kwamba tumepunguza idadi

kwa sababu ya moja.

47

00:04:17,500 --> 00:04:25,000

Donc F'= F - 1.

48

00:04:26,000 --> 00:04:33,400

Unajua hilo, kwa nadharia, tangu

A'=A-1 (yaani 12 hapa),

49

00:04:33,500 --> 00:04:40,000

umeonyesha uhusiano wa Euler kwa

grafu zote ambazo zina angalau kingo 12.

50

00:04:41,000 --> 00:04:47,000

Kwa hivyo unajua hii (S'-A'+F')

ni sawa na 2 kwa uhusiano wa Euler.

51

00:04:49,500 --> 00:04:51,900

Lakini sasa,

kwa kutumia mahusiano haya,

52

00:04:52,000 --> 00:04:55,700

unaweza pia kueleza S'-A'+F'

kulingana na S, A na F,

53

00:04:55,800 --> 00:04:59,000

ambazo ni nambari za wima, kingo

na nyuso za grafu ya mwanzo.

54

00:05:00,000 --> 00:05:13,000

Unaona hiyo S'-A'+F'=S-A+F (unaona

kwamba -1 wanalipana fidia).

55

00:05:15,000 --> 00:05:20,000

Na kwa hivyo unaona kwamba S-A+F=2:

hata hukuhitaji

56

00:05:20,050 --> 00:05:27,000

kuhesabu S-A+F kwa kuhesabu,

lakini ni lazima tu kujua hilo

57

00:05:27,100 --> 00:05:32,000

ilikuwa kweli kwa grafu hii kubaini

kwamba ilikuwa kweli kwa grafu hii.

58

00:05:36,000 --> 00:05:40,500

Huko, tulifanya moja ya uwezekano:

tulifuta makali haya.

59

00:05:40,600 --> 00:05:44,900

Lakini hii sio uwezekano pekee

inaweza kutokea, kwa mfano,

60

00:05:45,000 --> 00:05:51,000

ukifuta makali haya badala yake,

nini kitatokea tukiifuta?

61

00:05:51,100 --> 00:05:54,000

Kweli tunapata grafu hii,

62

00:05:54,500 --> 00:05:59,400

na unaona kwamba tunapata grafu

ambayo iko nje ya mtandao.

63

00:06:00,000 --> 00:06:04,000

Hiyo ni, unaweza kutenganisha

sehemu mbili,

64

00:06:04,100 --> 00:06:05,100

na unaweza kusema hapa,

65

00:06:05,150 --> 00:06:12,900

unayo graph nayo

Vipeo vya S1, kingo za A1, na nyuso za F1;

66

00:06:13,000 --> 00:06:19,000

et ici vous avez un autre graphe avec

Vipeo vya S2, kingo za A2, na nyuso za F2.

67

00:06:21,000 --> 00:06:22,900

Maintenant, ce que l'on veut,

68

00:06:23,000 --> 00:06:29,900

c'est exprimer S, A et F

en fonction de S1, S2, A1, A2, F1 et F2.

69

00:06:31,000 --> 00:06:38,900

Tout d'abord, S.

S est égal à quoi ?

70

00:06:39,000 --> 00:06:41,900

Je vous rappelle que S est le nombre de

sommets dans le graphe initial,

71

00:06:42,000 --> 00:06:44,900

avant qu'on ait effacé cette arête.

72

00:06:45,500 --> 00:06:50,000

Et bien vous voyez que S n'a pas bougé : on

n'a pas changé le nombre de sommets total.

73

00:06:51,500 --> 00:06:55,000

Ainsi, S est égal au nombre de sommets

dans ce graphe-ci, donc S1,

74

00:06:55,100 --> 00:06:58,500

plus le nombre de sommets

dans ce graphe-ci, donc S2.

75

00:06:59,000 --> 00:07:07,000

Donc S=S1+S2.

76

00:07:09,000 --> 00:07:11,000

Maintenant, pour le nombre d'arêtes.

77

00:07:12,000 --> 00:07:21,000

Vous savez que ces deux graphes-ci

ont été obtenus en effaçant une arête.

78

00:07:21,500 --> 00:07:28,900

Donc ça veut dire que A est égal à

ce nombre d'arêtes

79

00:07:29,000 --> 00:07:34,000

plus ce nombre d'arêtes plus 1,

puisqu'il y avait cette arête en plus.

80

00:07:34,100 --> 00:07:44,000

Donc c'est égal à A1+A2+1.

81

00:07:46,000 --> 00:07:52,000

Et enfin, F, le nombre de faces.

82

00:07:53,500 --> 00:07:58,000

On n'a pas changé le nombre de faces

à l'intérieur dans ce graphe-ci,

83

00:07:58,100 --> 00:08:03,000

ni dans ce graphe-là.

84

00:08:04,000 --> 00:08:08,900

En revanche vous voyez que maintenant ici

on a une face extérieure,

85

00:08:09,000 --> 00:08:13,000

et ici on en a une autre !

86

00:08:13,500 --> 00:08:17,900

Cela veut dire que, si vous comptez F1+F2,

87

00:08:18,000 --> 00:08:22,000

ça va être le nombre de faces

du graphe initial +1,

88

00:08:22,100 --> 00:08:25,000

puisqu'on compte la face extérieure

deux fois maintenant.

89

00:08:26,500 --> 00:08:32,500

F=F1+F2-1.

90

00:08:34,000 --> 00:08:39,500

Très bien, maintenant, calculons S-A+F.

91

00:08:40,000 --> 00:08:59,500

S-A+F= S1-A1+F1 + S2-A2+F2 -1-1.

92

00:09:29,500 --> 00:09:34,000

Ces deux-là, ça va faire -2.

93

00:09:36,000 --> 00:09:38,000

Maintenant, on sait que la relation d'Euler

94

00:09:38,100 --> 00:09:44,500

a été prouvée pour des graphes

avec un plus petit nombre d'arêtes que A.

95

00:09:44,600 --> 00:09:49,000

Donc pour ce graphe-ci par exemple,

avec un nombre d'arêtes A1,

96

00:09:49,100 --> 00:09:58,000

on sait que la relation d'Euler est vraie,

donc on sait que ceci (S1-A1+F1) vaut 2.

97

00:09:58,100 --> 00:10:01,500

De même, ici pour ce graphe,

on sait que la relation d'Euler est vraie,

98

00:10:01,600 --> 00:10:06,000

donc on sait que ceci (S2-A2+F2) vaut 2.

99

00:10:06,100 --> 00:10:17,000

Donc à la fin, on a que

S-A+F = 2+2-2 = 2.

100

00:10:17,100 --> 00:10:19,000

Et voilà !

101

00:10:19,500 --> 00:10:22,000

Voilà ! Merci d'avoir suivi cette vidéo !

102

00:10:22,500 --> 00:10:26,900

On a pu prouver la relation d'Euler

aujourd'hui pour les graphes planaires.

103

00:10:27,000 --> 00:10:32,000

Sachez que des formules similaires existent

aussi pour des graphes non planaires

104

00:10:32,100 --> 00:10:36,900

(ce sont des graphes où l'on peut

autoriser des croisements).

105

00:10:37,000 --> 00:10:40,000

Cette relation d'Euler est vraiment

universelle

106

00:10:40,100 --> 00:10:43,000

et c'est pour ça que je la trouve

très belle.

107

00:10:43,100 --> 00:10:46,000

Les chercheurs et les chercheuses

qui font de la combinatoire

108

00:10:46,100 --> 00:10:52,000

l'utilisent très souvent pour

classer les graphes qu'iels étudient.

109

00:10:53,000 --> 00:10:58,000

Merci beaucoup d'avoir suivi cette vidéo,

et à bientôt !